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Abelian projected $SU(2)$ Yang-Mills action for renormalisation group flows

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Abstract

The dual Meissner effect scenario of confinement is discussed by studying the low energy regime of $SU(2)$ Yang-Mills in a maximal Abelian gauge. The Abelian projected effective action is computed perturbatively. This serves as an input for a study of the non-perturbative regime, which is undertaken using exact renormalisation group methods. It is argued that the effective action derived here contains the relevant degrees of freedom for confinement if ultraviolet irrelevant vertices are retained.

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Introduction

The understanding of the low energy behaviour of QCD is one of the long standing challenges in theoretical particle physics. In this letter we elaborate on the dual Meissner mechanism for confinement [1] which is best studied in maximal Abelian gauges (MAGs). In these gauges the condensation of colour monopoles is explicit [2, 3]. 't Hooft [3] conjectured that these $U(1)$ -projected singularities govern confinement which sets the scenario of Abelian dominance.

This conjecture has been tested on the lattice. In gauge fixed lattice $SU(2)$ and $SU(3)$ strong evidence for Abelian dominance has been found [4]. It has been also shown that monopole condensates alone account for most of the string tension in the confining phase [5]. Other studies favour vortices as the relevant degrees of freedom [6]. Preliminary lattice results for the scale of Abelian dominance give $M_{\text{AD}} \simeq 1.2 \text{ GeV}$, clearly indicating that $M_{\text{AD}} > \Lambda_{\text{QCD}}$ [7]. Analytical aspects of Abelian dominance have been studied as well. This includes phenomenological models [8] and perturbative studies [9]. Moreover, a mass generating mechanism involving ghost pairs has been discussed by introducing a parallel with BCS superconductivity [10, 11].

The scale separation allows for a perturbative computation of the effective action Γ_Λ at scales $\Lambda \gg M_{\text{AD}}$. However, non-perturbative methods are needed at scales smaller than M_{AD} . The exact renormalisation group (ERG) for gauge theories is such a method [12, 13]. The ERG requires the effective action Γ_Λ as a key input and provides the ideal framework for investigating the above scenario. Its viability in the present context has already been highlighted in [14]. So far, the ERG has been formulated for general linear gauges [15]. However, the extension to the non-linear maximal Abelian gauges poses no new problems. One can easily transfer the result of studies carried out in the background field formalism [16].

In this letter, we compute an Abelian effective action for $SU(2)$ YM in a MAG. This is done by integrating over the charged gauge fields in the $SU(2)/U(1)$ coset broken by the MAG condition. All vertices up to the first UV non-relevant ones are kept. This provides us with an initial effective action for the ERG flow. We argue that this initial effective action contains the prerequisites for the dual Meissner effect: a coupling between a dual Abelian gauge field and a monopole current, and a kinetic term for the auxiliary tensor field. Finally, under certain conditions, we show the ERG flow drives the system into the confining phase.

The $SU(2)$ action

Let us begin by expressing the $SU(2)$ YM action in a four dimensional Euclidean space in

terms of a neutral and charged vector fields. In conventional variables the action is

$$S_{\text{YM}} = \frac{1}{4} \int_x F_{\mu\nu}^a F_{\mu\nu}^a, \quad (1)$$

where \int_x is a short hand for $\int d^4x$ and the field strength is $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon^{abc} A_\mu^b A_\nu^c$, with g the gauge coupling parameter and ϵ^{abc} the canonical antisymmetric 3-tensor. We now introduce the new field variables

$$A_\mu = A_\mu^3, \quad \phi_\mu = \frac{1}{\sqrt{2}} (A_\mu^1 - iA_\mu^2), \quad \phi_\mu^\dagger = \frac{1}{\sqrt{2}} (A_\mu^1 + iA_\mu^2). \quad (2)$$

These are easily recognised as the gauge field components associated with the diagonal $SU(2)$ generator and the off-diagonal lowering and raising ones respectively.

When expressed in terms of the fields (2) the YM action takes the form

$$S_{\text{YM}} = \frac{1}{4} \int_x (f_{\mu\nu} + C_{\mu\nu}) (f_{\mu\nu} + C_{\mu\nu}) + \frac{1}{2} \int_x \Phi_{\mu\nu}^\dagger \Phi_{\mu\nu}, \quad (3)$$

where $f_{\mu\nu}$ and $\Phi_{\mu\nu}$ are the field strengths for the Abelian and the charged vector fields, respectively, given by $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\Phi_{\mu\nu} = D_\mu \phi_\nu - D_\nu \phi_\mu$, with $D_\mu = \partial_\mu - igA_\mu$ the Abelian covariant derivative. Moreover, $C_{\mu\nu}$ in (3) is a real, antisymmetric, quadratic combination of the off-diagonal fields,

$$C_{\mu\nu} = ig (\phi_\mu^\dagger \phi_\nu - \phi_\nu^\dagger \phi_\mu). \quad (4)$$

The YM action (3) is explicitly invariant under a $U(1)$ subgroup of $SU(2)$, where under $U(1)$ the charged vector fields are transformed only by an Abelian phase factor. Of course the action is $SU(2)$ invariant as well. There are two types of quartic coupling in (3). Those between the neutral and the charged vector fields, $A_\mu A_\mu \phi_\nu^\dagger \phi_\nu - A_\mu A_\nu \phi_\mu^\dagger \phi_\nu$, and the charged vector fields self-couplings,

$$\frac{1}{4} C_{\mu\nu} C_{\mu\nu} = -\frac{1}{2} g^2 (\phi_\mu^\dagger \phi_\nu \phi_\mu^\dagger \phi_\nu - \phi_\mu^\dagger \phi_\nu \phi_\nu^\dagger \phi_\mu). \quad (5)$$

Hitherto, we have restricted the discussion to the classical level. The quantisation of YM theories is non-trivial because of the constraints. Besides, if we insist on a covariant formulation we need to handle unphysical zero modes. It is well known how to treat this problem. Here we pursue the path integral quantisation with gauge fixing and ghost fields.

The action (3) is well-suited to study Abelian dominance in the continuum because of its explicit $U(1)$ invariance. For the gauge-fixing we choose a MAG condition which leaves the $U(1)$ invariance in (3) unbroken. Then, the remnant gauge freedom is fixed with a Lorentz condition, i.e. respectively

$$F^\pm[\phi, A] := (\partial_\mu \pm igA_\mu) \phi_\mu = 0, \quad F[A] := \partial_\mu A_\mu = 0. \quad (6)$$

Though Gribov copies exist in a MAG they do not give rise to any sizable effects [17] and can be ignored. Then the gauge fixing sector including the ghost action is

$$S_g = \frac{1}{2\xi} \int_x (\partial_\mu A_\mu)^2 + \frac{1}{\xi'} \int_x (D_\mu \phi_\mu)^\dagger (D_\nu \phi_\nu) - \int_x \bar{c}_+ (D_\mu^\dagger D_\mu^\dagger + g^2 \phi_\mu^\dagger \phi_\mu) c_+ - \\ - \int_x \bar{c}_- (D_\mu D_\mu + g^2 \phi_\mu^\dagger \phi_\mu) c_- + g^2 \int_x \bar{c}_+ c_- \phi_\mu^\dagger \phi_\mu + g^2 \int_x \bar{c}_- c_+ \phi_\mu \phi_\mu. \quad (7)$$

The quantum theory for the gauge fixed action $S = S_{\text{YM}} + S_g$ will now be studied in a path integral representation.

Abelian dominance

The suppression of the ϕ_μ fields [4] suggests that they acquire a mass [7, 10, 11] dynamically, but this mass cannot be computed perturbatively due to BRS invariance. We expect that this mass sets the scale of Abelian dominance, M_{AD} , below which a qualitative change of the relevant dynamical variables takes place. At a much lower scale this will lead to confinement. Preliminary results from the lattice give $M_{\text{AD}} \simeq 1.2 \text{ GeV}$ [7]. Though we expect that this value might decrease, or that for $SU(3)$ it should be smaller, it is an indication that the scale of Abelian dominance is larger than the confining scale Λ_{QCD} .

The first step towards an effective Abelian theory is to integrate over the charged vector fields. The presence of vertices with four of these fields (5) hinders a straightforward integration. This problem can be surmounted with the introduction of an auxiliary tensor field, $B_{\mu\nu}$.

The tensor field is introduced in (3) via the replacement,

$$\frac{1}{4} \int_x C_{\mu\nu} C_{\mu\nu} \longrightarrow -\frac{1}{4} \int_x \tilde{B}_{\mu\nu} \tilde{B}_{\mu\nu} + \frac{1}{2} \int_x \tilde{B}_{\mu\nu} C_{\mu\nu}. \quad (8)$$

It follows that the equation of motion for $B_{\mu\nu}$ is $\tilde{B}_{\mu\nu} = C_{\mu\nu}$. After inserting it back into (8), the term quadratic in $C_{\mu\nu}$ is recovered. When performing the replacement (8) in the full action S the integration over the charged vector fields becomes Gaussian. Now we focus our attention on the part of the action quadratic in these fields. We get

$$S_{\phi^2} = \int_x \left(\phi_\mu^\dagger a_{\mu\nu}^{-+} \phi_\nu + \phi_\mu a_{\mu\nu}^{+-} \phi_\nu^\dagger + \phi_\mu a_{\mu\nu}^{++} \phi_\nu + \phi_\mu^\dagger a_{\mu\nu}^{--} \phi_\nu^\dagger \right), \quad (9)$$

where the elements of the rank two matrix $a_{\mu\nu}^{SS'}$, with $s, s' = \pm$, are

$$a_{\mu\nu}^{-+} = -\frac{1}{2} g_{\mu\nu} (D_\rho D_\rho + g^2 \bar{c}_+ c_+) + \frac{1}{2\xi'} (\xi' - 1) D_\mu D_\nu + \frac{ig}{2\xi'} [(\xi' + 1) f_{\mu\nu} + \xi' \tilde{B}_{\mu\nu}], \\ a_{\mu\nu}^{+-} = [a_{\mu\nu}^{-+}]^\dagger, \quad a_{\mu\nu}^{++} = g^2 g_{\mu\nu} \bar{c}_- c_+, \quad a_{\mu\nu}^{--} = [a_{\mu\nu}^{++}]^\dagger. \quad (10)$$

From now on, for simplicity, we take $\xi' = 1$.

Due to the Gaussian integration the effective action receives a contribution of the form, $\frac{1}{2} \text{Tr} \ln a_{\mu\nu}^{SS'}$. In a Schwinger proper-time representation it reads

$$\frac{1}{2} \text{Tr} \ln (a_{\mu\nu}^{SS'}) = -\frac{1}{2} \lim_{s \rightarrow 0} \frac{d}{ds} \left(\frac{\mu^{2s}}{\Gamma(s)} \int_0^\infty dt t^{s-1} \text{Tr} \left[e^{-ta_{\mu\nu}^{SS'}} - e^{-ta_0^{SS'}} \right] \right), \quad (11)$$

where $a_{o\mu\nu}^{SS'}$ is $a_{\mu\nu}^{SS'}$ at vanishing fields and a_0 is introduced as a regulator. The trace Tr is over covariant and group indices and a complete set of states. Finally, the constant μ is an arbitrary regulator scale. The part of the trace corresponding to the sum over a complete set of states takes a very convenient integral form for a complete base of plane waves, which will be very suitable to develop systematic approximations. Together with the change of variable, $k_\rho \rightarrow \sqrt{t} k_\rho$, the trace written in a base of plane waves is now

$$\text{Tr} \left[e^{-ta_{\mu\nu}^{SS'}} \right] = \frac{t^{-2}}{(2\pi)^4} \int_x \int_k e^{-\frac{1}{2}k^2} \text{tr} \left[\exp \left[-t a_{\mu\nu}^{SS'} + i\sqrt{t} g_{\mu\nu} \left(\bar{\delta}^{S+} \bar{\delta}^{S'-} k_\rho D_\rho + \bar{\delta}^{S-} \bar{\delta}^{S'+} k_\rho D_\rho^\dagger \right) \right] \right], \quad (12)$$

where tr is the trace restricted to the covariant and group indices. We have also introduced the inverse Kronecker delta, $\bar{\delta}^{SS'} = 1$ if $s \neq s'$ and $\bar{\delta}^{SS'} = 0$ if $s = s'$. After inserting (12) into the right-hand side of (11) and Taylor expanding the exponentials, the contribution to the effective action becomes

$$\begin{aligned} \frac{1}{2} \text{Tr} \ln (a_{\mu\nu}^{SS'}) = & -\frac{1}{2} \lim_{s \rightarrow 0} \frac{d}{ds} \frac{\mu^{2s}}{\Gamma(s)} \int_0^\infty dt t^{s-3} \int_x \sum_{n=0}^\infty \frac{(-1)^n}{n!} \int_k e^{-\frac{1}{2}k^2} \times \\ & \text{tr} \left[\left(t a_{\mu\nu}^{SS'} - i\sqrt{t} g_{\mu\nu} \left(\bar{\delta}^{S+} \bar{\delta}^{S'-} k_\rho D_\rho + \bar{\delta}^{S-} \bar{\delta}^{S'+} k_\rho D_\rho^\dagger \right) \right)^n - \left(t a_0^{SS'} - i\sqrt{t} g_{\mu\nu} \bar{\delta}^{SS'} k_\rho \partial_\rho \right)^n \right]. \end{aligned} \quad (13)$$

In order to progress beyond Eq. (13), it is necessary to employ approximations. The momentum integration is convergent but not solvable. We will proceed with an expansion on small t which provides a systematic approach of computing the effective vertices for short range interactions by decreasing importance in their UV relevance.

Ultraviolet relevance

On the right-hand side of (13) only the integer powers of the Schwinger proper-time do not vanish after the integration over the momenta, because the integrand for the half integer powers of t is odd in the momenta. The integration in the Schwinger proper-time is now splitted into two part, $\int_0^{1/\Lambda^2} + \int_{1/\Lambda^2}^\infty$. The scale Λ is a UV scale larger than M_{Ad} chosen so that the integration for $t < 1/\Lambda^2$ is negligible due to Abelian dominance. Therefore, we keep only the second part of the integration with $1/\Lambda^2$ in the lower bound. Since we are interested in the leading UV vertices only the contributions coming from this bound are retained. After the integration, the expansion in powers of t emerges as an expansion in

vertex operators of decreasing UV relevance. Here we keep the terms of this expansion up to the first non-relevant ones, i.e. $\mathcal{O}(1/\Lambda^2)$. The resulting effective action, with $\mu = \Lambda$, is

$$\begin{aligned}
S_{\text{eff}} = & \frac{1}{4} \left(1 + g^2 \frac{5\gamma}{12\pi^2} \right) \int_x f_{\mu\nu} f_{\mu\nu} + \frac{1}{4} \int_x B_{\mu\nu} \left(\frac{g^2}{96\pi^2} \frac{\square}{\Lambda^2} - 1 + g^2 \frac{\gamma}{8\pi^2} \right) B_{\mu\nu} + \\
& + g^2 \frac{\gamma}{8\pi^2} \int_x \tilde{f}_{\mu\nu} B_{\mu\nu} - \int_x \bar{c}_+ \left(D_\mu^\dagger D_\mu^\dagger + \frac{g^2}{2\pi^2} \Lambda^2 \right) c_+ - \int_x \bar{c}_- \left(D_\mu D_\mu + \frac{g^2}{2\pi^2} \Lambda^2 \right) c_- - \\
& - g^4 \frac{\gamma}{2\pi^2} \int_x \bar{c}_+ c_+ \bar{c}_- c_- + \frac{1}{2\xi} \int_x (\partial_\mu A_\mu)^2 + \\
& + \frac{1}{\Lambda^2} \left(\frac{g^2}{96\pi^2} \int_x \tilde{f}_{\mu\nu} \square B_{\mu\nu} + \frac{11g^2}{960\pi^2} \int_x f_{\mu\nu} \square f_{\mu\nu} + \frac{g^4}{192\pi^2} \int_x (\bar{c}_+ c_+ + \bar{c}_- c_-) B_{\mu\nu} B_{\mu\nu} + \right. \\
& + \frac{g^4}{48\pi^2} \int_x (\bar{c}_+ c_+ + \bar{c}_- c_-) \tilde{f}_{\mu\nu} B_{\mu\nu} + \frac{g^4}{96\pi^2} \int_x (\bar{c}_+ c_+ + \bar{c}_- c_-) f_{\mu\nu} f_{\mu\nu} + \\
& \left. + \frac{g^4}{48\pi^2} \int_x (\bar{c}_- c_+) D_\mu^{(2)} D_\mu^{(2)} (\bar{c}_+ c_-) \right) + \mathcal{O} \left(\frac{1}{\Lambda^4}, B^4 \right), \tag{14}
\end{aligned}$$

with $D_\mu^{(2)} = \partial_\mu - 2igA_\mu$, the covariant derivative for a charged two field. Note that the $\mathcal{O}(\Lambda^2)$ terms correspond to a mass renormalisation, and the coupling and wave function renormalisation are identified in (14) as the terms proportional to the Euler gamma. The latter is proportional to $(\ln[\Lambda/\mu] - \gamma/2)$ but we have set the renormalisation constant scale μ in (11) equal to Λ . We note that in a more complete treatment where the hard modes of the remaining fields are integrated out to one-loop order down to Λ , the $\mathcal{O}(\Lambda^2)$ ghost terms are cancelled in accordance to BRS invariance. Finally, as expected, the effective action (14) is $U(1)$ invariant.

The UV marginal terms account for wave function and coupling renormalisations. There are two new vertices that were not present at tree level: the coupling between the dual of the Abelian field strength and the tensor field, and the 4-ghosts vertex, the first terms in the second and third line of (14) respectively. As far as monopoles are concerned, the relevant vertex appearing at this order is the one involving the auxiliary tensor field $B_{\mu\nu}$. Kondo [9] has shown by using a Hodge decomposition of $B_{\mu\nu}$ that the $\tilde{f}_{\mu\nu} B_{\mu\nu}$ term encapsulates the coupling between a gauge field potential and a magnetic current $J_\nu^M = \partial_\mu \tilde{f}_{\mu\nu}$. Note, that no term involving $f_{\mu\nu} B_{\mu\nu}$ is generated at this order. At present, we do not have a clear explanation for it, but its existence seems to imply that instead of a condensate of monopoles, we might have a condensate of dyons.

Finally, we comment on the first correction of UV irrelevant vertices. One vertex was singled out in (14), namely the term governing the dynamics of the tensor field. This term was placed in the first line of the right-hand side of (14). The remaining $\mathcal{O}(1/\Lambda^2)$ terms are displayed in the last three lines of (14). The irrelevant vertex, included in the leading UV terms, corresponds to the leading ghost free term in lowest order in a derivative expansion. Below, we will discuss this operator at greater length.

A few remarks on the scales μ and Λ should be made at this stage. The scale μ is the renormalisation group scale coming from the dimensional regularisation in the Schwinger proper-time representation (11). In order to define the effective theory a UV scale Λ is introduced and the resulting one-loop logarithms are of the form $\ln[\Lambda/\mu]$ as referred above. The choice $\mu = \Lambda$ is naturally a convenient one. The resulting effective action, (14), is in the spirit of [18] an appropriate initial condition at the scale Λ for an ERG analysis of the low energy theory.

Information on the relative magnitudes of the scale Λ to M_{AD} is necessary to complete this discussion. The sole requirement so far on Λ was that it is in the perturbative region of $SU(2)$ YM. This ensures that the coefficients of the different vertices can be determined reliably about this UV scale. However, in the present problem we expect two more, though related, scales: the confinement scale Λ_{QCD} and the scale of Abelian dominance M_{AD} , with $M_{\text{AD}} > \Lambda_{\text{QCD}}$. In order to study how the presence of M_{AD} might condition the choice of Λ we have also calculated the effective action when the charged vector fields have a mass M_{AD} inserted in (10). We found that the coefficients in (14) receive corrections in a power series with respect to $M_{\text{AD}}^2/\Lambda^2$. This indicates that in order for (14) to provide a reliable effective action we need to require $\Lambda \gg M_{\text{AD}}$. Therefore, we can safely expect to improve the computation of (14) within the presently available methods.

Exact renormalisation group and confinement

The main goal here is to show that qualitatively the Abelian effective action (14) contains the relevant degrees of freedom for confinement. Therefore, in the following we ignore the ghost sector and only keep terms up to quadratic order in a derivative expansion. The effective action is used as the initial condition to the ERG equations at the scale Λ .

The ERG equations are flow equations for the effective action Γ_k with respect to the scale k running from $k = \Lambda$ to $k = 0$. The k dependence comes from the insertion of IR cut-off functions R_k in Γ_k (for the gauge and ghost fields in the present case). The flow equation has the structure of a one-loop equation

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left[\partial_t R_k \left(\Gamma_k^{(2)} + R_k \right)^{-1} \right], \quad t = \ln k, \quad (15)$$

where Tr stands for the sum over all fields and group indices as well as the integration over

configuration (or momentum) space, whilst $\Gamma_k^{(2)}$ is the fully dressed 1PI functional [12]. Diagrammatically, the right-hand side of Eq. (15) is the sum of one-loop diagrams with a $\partial_t R_k$ insertion and full propagators.

For the problem discussed in this letter, we have seen that the Abelian effective action (14) contains the leading and near-to-leading UV operators. Therefore, for the effective action Γ_k we choose as an Ansatz,

$$\Gamma_k = \int_x \left\{ \frac{Z_A}{4} f_{\mu\nu} f_{\mu\nu} + \frac{1}{4} (-Z_B \square + M_B^2) B_{\mu\nu} B_{\mu\nu} + \frac{Y}{2} \tilde{f}_{\mu\nu} B_{\mu\nu} + \frac{1}{2\xi} (\partial_\mu A_\mu)^2 + \text{ghosts} \right\}. \quad (16)$$

The coefficients Z_A , Z_B , M_B^2 and Y are renormalisation functions which depend on k . The vertices in this Ansatz consist of the first order terms in (14) plus the leading $\mathcal{O}(1/\Lambda^2)$ term in a derivative expansion. The dynamics of the Abelian gauge field and the tensor field are governed by the first two terms respectively. The third term establishes the coupling between these two fields. Effectively it couples the magnetic current to a dual magnetic field [9]. The fourth term is the gauge fixing and “ghosts” stands for all the terms involving ghosts up to the first order in derivative expansion. These terms are left out as their presence would not have altered the results presented below. In any case, as we have an effective Abelian theory, they should not play a major role for the leading structure of the gauge field propagator.

Next, we show that this Ansatz contains enough information to exhibit a confining phase at low energies. As a signature of confinement we search for a $1/p^4$ singularity in the gauge field propagator. It is well established that a propagator with this feature leads to an area law in a Wilson loop [19]. By implementing the Ansatz (16) the problem of finding the flow of the functional Γ_k is reduced to that of determining the running of Z_A , Z_B , M_B^2 and Y as functions of k .

The initial condition for the ERG flow is assigned by taking $\Gamma_{k=\Lambda} \approx S_{\text{eff}}$ which, in particular, for the new coefficient Z_B reduces to $Z_B(\Lambda) \approx 0$. Of course, this simply reflects that $B_{\mu\nu}$ was introduced as an auxiliary field. From S_{eff} in (14) we have that $M_B^2 < 0$ at the initial scale. However the coefficient of any vertex of higher power in the tensor field $B_{\mu\nu}$ can be shown to be positive. It seems to indicate that $B_{\mu\nu}$ acquires a VEV but most importantly it guarantees the stability of the effective action along the tensor field direction. Therefore, by taking $B_{\mu\nu}$ to be the fluctuation field about its global minimum we guarantee M_B^2 to be positive. This will suffice for our present purposes.

A useful feature of working with an Ansatz as (16) is that propagators can be expressed as functionals of the renormalisation functions for any value of $k \leq \Lambda$. The functional form of the propagators is determined by simply inverting the two point 1PI Green’s functions.

Then, the gauge field propagator of Γ_k as given in (16), is

$$(P_{AA})_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z_B p^2 + M_B^2}{Z_A Z_B p^4 + p^2 (M_B^2 Z_A - Y^2)} + \xi \frac{p_\mu p_\nu}{p^4}. \quad (17)$$

In the Landau gauge, $\xi = 0$, we observe that the denominator of the gauge field propagator (17) is dominated by the p^4 term if

$$M_B^2(k) Z_A(k) = Y(k)^2 \quad (18)$$

is an IR stable quasi-fixed point. Then, if the condition (18) is realised, the gauge field propagator will have a $1/p^4$ behaviour at small momentum, if the p dependence on the numerator becomes negligible, i.e.

$$Z_B(k_c) p^2 \ll M_B^2(k_c) \quad \Rightarrow \quad \sqrt{p^2} \ll \frac{M_B(k_c)}{\sqrt{Z_B(k_c)}} := \Lambda_{\text{QCD}}. \quad (19)$$

This result should be discussed in parallel to the findings of Ellwanger [14]. Our treatment differs in three main aspects: (a) we work in a MAG instead of the Landau gauge; (b) we present a systematic computation of the action on which we base our Ansatz. Ellwanger's Ansatz is based on the requirement of full BRS invariance and the Abelian projection follows as a truncation to a diagonal Abelian component. Our systematic calculation provides a more direct link with the parameters of the theory; (c) the tensor field used by Ellwanger is the dual of the entire field strength of the 't Hooft-Polyakov monopole [20] while we consider only the dual of its quadratic part.

In his work, Ellwanger found a condition analogous in form to (18). Furthermore, evidence for a quasi-fixed point was found within his approximations. Further investigation is required to verify, whether the same applies in the present case. However, in view of the similarities we expect an analogous outcome. More recently, Ellwanger and Wschebor [21], have found that for an effective gauge theory where it is assumed that the charged fields as well as the ghosts have been integrated over, the equivalent to condition (18) is relaxed to an inequality which in the present case would read $M_B^2(k) Z_A(k) - Y(k)^2 < 0$, for $k > 0$. The equality is recovered in the IR limit $k \rightarrow 0$.

Confinement and the dynamics of the $B_{\mu\nu}$ field

We have seen that the propagator of the gauge field can, under certain conditions, lead to a linear effective quark potential. The IR $1/p^4$ singularity occurs because: (a) there is a term proportional to p^4 in the denominator of the propagator that becomes prominent when the condition (18) holds; (b) at small momentum, when (19) is fulfilled, the numerator becomes p^2 independent. The proportionality factor to p^4 in the denominator is $Z_A Z_B$ where Z_A is

always non-zero as the $U(1)$ gauge field remains dynamical in any phase. However, the situation is different for Z_B . The tensor field is not dynamical at short distances, $Z_B \approx 0$, which reflects the UV irrelevance of the $B_{\mu\nu}$ field kinetic term.

Therefore in order to have a non-vanishing factor $Z_A Z_B \not\approx 0$, somewhere in between the deep UV region $k \simeq \Lambda$ and the confinement scale Λ_{QCD} , the tensor field kinetic vertex must undergo a crossover that will make it relevant in the IR. We expect this crossover scale to be linked to the scale of Abelian dominance M_{AD} . Above M_{AD} the dynamics of $B_{\mu\nu}$ is protected by the still unsuppressed off-diagonal gauge fields by a $\mathcal{O}(1/\Lambda^2)$ factor. Below M_{AD} the effects of gauge fields associated with the off-diagonal components quickly lose prominence. This is counterbalanced by terms involving the $U(1)$ invariant tensor field. Consequently, Z_B plays an equally relevant role in the Abelian dominated regime when compared with the other renormalisation functions. Hence, we can conclude from (17) that the scenario described above leading to a $1/p^4$ behaviour is plausible. Polonyi's view of confinement as an irrelevant-to-relevant crossover [22] of some UV irrelevant operator is in line with the present scenario.

Summary and discussion

We have combined the benefits of working in a MAG and using ERG methods to study the monopole mechanism for confinement. The present approach is based on the assumption that an intermediate Abelian dominance scale M_{AD} is dynamically generated, a viewpoint supported by lattice results. Starting from a pure $SU(2)$ YM theory an Abelian effective action was derived. In the calculation we used a maximal Abelian gauge, introduced an auxiliary tensor field, and integrated over the gauge fields in $SU(2)/U(1)$. This serves as an initial effective action for an ERG flow. The initial UV scale Λ is well inside the perturbative region of the theory. It is argued that confinement occurs if the ERG flow at low energies settles about an IR stable quasi-fixed point. Here, confinement hinges on an irrelevant-to-relevant crossover for the kinetic term of the auxiliary tensor field. We expect the crossover scale to be the scale of Abelian dominance M_{AD} .

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